3.8) $B = \{ u_1, u_2, u_3 \}$ con $\|u_1\|, \|u_2\|, \|u_3\| = 1$

a) Sean x e $y \in V$ y B base de V:

$$x = \alpha_1 \cdot u_1 + \alpha_2 \cdot u_2 + \alpha_3 \cdot u_3$$

$$y = \gamma_1 \cdot u_1 + \gamma_2 u_2 + \gamma_3 u_3$$

donde $[x]_B = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$ y $[y]_B = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}$

(TENGO EN CUENTA QUE SON REALES P/APLICAR LAS PROP. DE PI)

$$\to (x,y) = \left(\sum_{i=1}^{3} \alpha_i . u_i , \sum_{j=1}^{3} \gamma_j u_j \right) = \left(\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 , \gamma_1 u_1 + \gamma_2 u_2 + \gamma_3 u_3 \right) = \to$$

PROP. PI

$$= \sum_{i=1}^{3} \left(\alpha_i . u_i , \sum_{j=1}^{3} \gamma_j u_j \right) = \longrightarrow$$

PROP. PI

$$\to = \sum_{i=1}^{3} \alpha_i \left(u_i , \sum_{j=1}^{3} \gamma_j u_j \right) \overset{\text{PROP. PI}}{=} \sum_{i=1}^{3} \alpha_i \sum_{j=1}^{3} \left(u_i , \gamma_j u_j \right) = \longrightarrow$$

$$\xrightarrow{} \overset{\text{Prop. PI}}{=} \sum_{i=1}^{3} \alpha_i \sum_{j=1}^{3} \bar{\gamma}_i (v_i, v_j) \overset{\text{Prop. sumatoria}}{=} \sum_{i=1}^{3} \sum_{j=1}^{3} \alpha_i \bar{\gamma}_i (v_i, v_j)$$

donde la matriz del PI o de Gram es:

$$G_B = \begin{bmatrix} (U_1, U_1) & (U_1, U_2) & (U_1, U_3) \\ (U_2, U_1) & (U_2, U_2) & (U_2, U_3) \\ (U_3, U_1) & (U_3, U_2) & (U_3, U_3) \end{bmatrix}$$

Usando los datos del enunciado y la identidad de Polarización:

$$(U_1, U_1) = (U_2, U_2) = (U_3, U_3) = \sqrt{1} = 1$$

$$\underset{i \neq j}{(U_i, U_j)} = \frac{1}{4}\left(2 + \sqrt{3} - 2 + \sqrt{3}\right) = \frac{\sqrt{3}}{2}$$

$$\rightarrow \boxed{G_B = \begin{bmatrix} 1 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}}$$

6) $\Theta = \left[\arccos(\langle u_i, u_j \rangle)\right]_{\substack{i \in I_3 \\ j \in I_3}}$

$$\Theta = \begin{bmatrix} \arccos(\langle v_1, v_1 \rangle) & \arccos(\langle v_1, v_2 \rangle) & \arccos(\langle v_1, v_3 \rangle) \\ \arccos(\langle v_2, v_1 \rangle) & \arccos(\langle v_2, v_2 \rangle) & \arccos(\langle v_2, v_3 \rangle) \\ \arccos(\langle v_3, v_1 \rangle) & \arccos(\langle v_3, v_2 \rangle) & \arccos(\langle v_3, v_3 \rangle) \end{bmatrix}$$

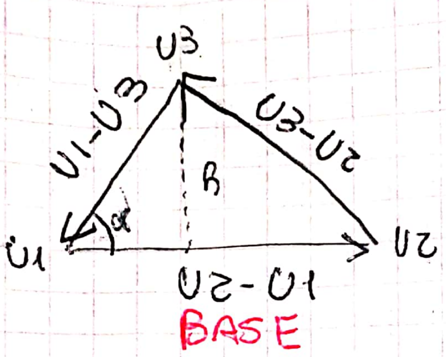
$\arccos(\langle v_1, v_1 \rangle) = \arccos(1) = 0 = \arccos(\langle v_2, v_2 \rangle) = \arccos(\langle v_3, v_3 \rangle)$

$\arccos(\langle v_1, v_2 \rangle) = \arccos(\langle v_2, v_1 \rangle) = \arccos\left(\frac{\sqrt{3}}{2}\right) = \pi/6$

$\arccos(\langle v_1, v_3 \rangle) = \arccos(\langle v_3, v_1 \rangle) = \arccos\left(\frac{\sqrt{3}}{2}\right) = \pi/6 = \arccos(\langle v_2, v_3 \rangle) = \arccos(\langle v_3, v_2 \rangle)$

$$\rightarrow \boxed{\Theta = \begin{bmatrix} 0 & \pi/6 & \pi/6 \\ \pi/6 & 0 & \pi/6 \\ \pi/6 & \pi/6 & 0 \end{bmatrix}}$$

8c)



Pon enunciado.

La base es:

$\|U_2-U_1\| \to \|U_2-U_1\|^2 = 2-\sqrt{3} \to \boxed{\|U_2-U_1\| = \sqrt{2-\sqrt{3}}}$

BASE

$\|U_1-U_3\| \to \|U_1-U_3\|^2 = 2-\sqrt{3} \to \|U_1-U_3\| = \sqrt{2-\sqrt{3}}$

$(U_2-U_1, U_1-U_3) = (U_2,U_1) - (U_2,U_3) - \|U_1\|^2 + (U_1,U_3)$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - 1 + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}-2}{2}$

Entonces:

Cambio signo por sentido del vector.

$$\cos\alpha = \frac{(U_2-U_1, -U_1+U_3)}{\|U_2-U_1\| \, \underbrace{\|-U_1+U_3\|}_{=\|U_1-U_3\|}} = \frac{\frac{2-\sqrt{3}}{2}}{7-4\sqrt{3}} = \frac{2-\sqrt{3}}{14-8\sqrt{3}}$$

$\rightarrow \alpha =$

Entonces:

Cambio signo por sentido del vector

$$\cos\alpha = \frac{(U_2-U_1, -U_1+U_3)}{\|U_2-U_1\| \, \underbrace{\|-U_1+U_3\|}_{=\|U_1-U_3\|}} = \frac{\frac{2-\sqrt{3}}{2}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-2\sqrt{3}}$$

$$\rightarrow \boxed{\alpha = \frac{\pi}{3}}$$

Por lo tanto, la altura es: $h = \|U_1-U_3\| . \operatorname{sen}\alpha$

$$\rightarrow \boxed{h = \sqrt{2-\sqrt{3}} \, . \operatorname{sen}\left(\frac{\pi}{3}\right)}$$

ALTURA

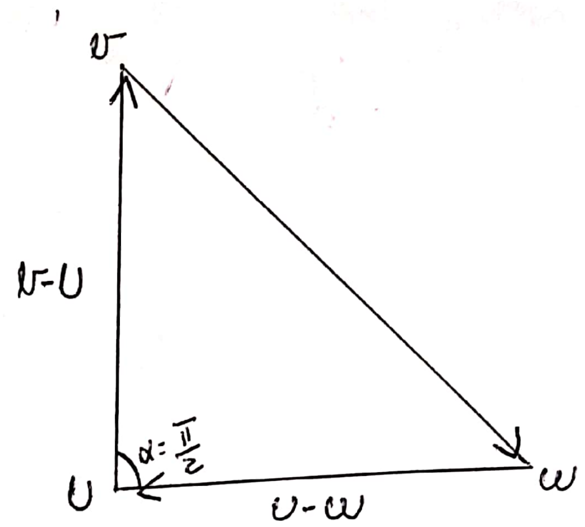
Por lo tanto, ahora puedo calcular el área:

$$A = \frac{\text{base} . \text{altura}}{2} = \frac{\sqrt{2-\sqrt{3}} \, . \, \sqrt{2-\sqrt{3}} \, . \operatorname{sen}(\pi/3)}{2}$$

$$\rightarrow \boxed{A = \frac{(2-\sqrt{3}) \operatorname{sen}(\pi/3)}{2} = 0{,}116}$$

ÁREA

d)



Quiero que $\boxed{\|v-u\| = 3}$ y $\boxed{\|u-w\| = 4}$.

y que $\boxed{(v-u, w-u) = 0}$

Puedo suponer $u=0$, $v = \alpha u_1$, $w = \gamma u_1 + \theta u_2$

de manera que $u, v, w \in \text{gen}\{u_1, u_2\}$.

Busco los escalares:

$$\|v-u\| = 3 \to \|\alpha u_1\| = 3 \to |\alpha|\,\|u_1\| = 3 \to |\alpha| = 3$$

Puedo tomar $\alpha = 3$ o $\alpha = -3$, tomo $\boxed{\alpha = 3}$.

$$(v-u, w-u) = 0 \to (v, w) = 0 \to (3u_1, \gamma u_1 + \theta u_2) = 0$$

$$\to 3\gamma(u_1, u_1) + 3\theta(u_1, u_2) = 3\gamma + 3\theta \cdot \frac{\sqrt{3}}{2}$$

$$\to \gamma = -\theta \frac{\sqrt{3}}{2}$$

$$\|u-w\| = \|w-u\| = 4 \to \|\gamma u_1 + \theta u_2\| = 4 \to \overbrace{\|\gamma u_1 + \theta u_2\|^2}^{I} = 16 \quad (\Delta)$$

I $\rightarrow \|\gamma u_1 + \theta u_2\|^2 = (\gamma u_1 + \theta u_2, \gamma u_1 + \theta u_2)$

$= \gamma^2 \cdot \|u_1\|^2 + \gamma\theta \cdot (u_1, u_2) + \gamma\theta \cdot (u_2, u_1) + \theta^2 \cdot \|u_2\|^2$

$= \gamma^2 \cdot \|u_1\|^2 + 2\gamma\theta (u_1, u_2) + \theta^2 \cdot \|u_2\|^2$

$= \gamma^2 + \theta^2 + \gamma\theta \cdot \sqrt{3}$

$\overset{\Delta}{=} \theta^2 \cdot \frac{3}{4} + \theta^2 - \theta^2 \cdot \frac{3}{2} = \theta^2 \cdot \underbrace{\left(\frac{3}{4} + 1 - \frac{3}{2}\right)}_{= \frac{1}{4}} = 16$

$\rightarrow \theta^2 = 16 \cdot 4$

$\rightarrow |\theta| = \sqrt{64} \rightarrow |\theta| = 8$

Puedo tomar $\theta = 8$ y $\theta = -8$, tomo $\boxed{\theta = 8}$

Entonces $\gamma = -8 \frac{\sqrt{3}}{2} \rightarrow \boxed{\gamma = -4\sqrt{3}}$

Por lo tanto el triángulo pedido tiene vértices:

$\boxed{0,\ 3u_1 \text{ y } -4\sqrt{3}u_1 + 8u_2}$

VERTICES